ground water

Computer Note/

A Procedure for Automated Analysis of Brief Pumping Tests of Domestic Wells

by Kate Klusman[1]

Abstract

A new computer program has been developed to automate analysis of brief single-well pumping tests. Adapted from a procedure developed by Picking (1994) that does not require measurement of the pumping rate, this new program is menu-driven and eliminates one significant source of imprecision in Picking's original method, namely, selection of "well function of u" values by interpolation in a lookup table. This new program has been applied to tests of 25 domestic wells penetrating bedrock, each pumped for < 2 min.

Theory

Picking (1994) presented a procedure for calculating aquifer transmissivity and storage by analyzing water level recovery in a pumped well following a brief period of pumping at an unknown constant rate. The only data required are the well radius, the times when pumping started and stopped, and a series of water level measurements at known times thereafter.

Innumerable reports have followed Theis (1935) in expressing transient drawdown around a pumped well as

$$Y = Q\,W(u)/4\pi T \tag{1}$$

where Y is drawdown at a distance, r, from the well centerline at an elapsed time, t, since pumping began at a constant rate, Q; T is transmissivity; W(u) is a complex expression termed the "well function of u" with u being $r^2S/4Tt$; and S is storage coefficient (or storativity).

Theis assumed that well radius was small enough that storage in the wellbore could be neglected. Later, Papadopulos and Cooper (1967) developed the following equation in which F(u, α) is a substitute for W(u) that allows Equation 1 to describe drawdown in the pumped well while accounting for storage in the wellbore:

$$F(u,\alpha) = \frac{32\alpha^2}{\pi^2}\int_0^\infty \frac{1 - e^{-\beta^2/4u}}{\beta^3\Delta(\beta)}d\beta \tag{2}$$

where β is Tt/r^2; Δ(β) is $[\beta J_0(\beta) - 2\alpha J_1(\beta)]^2 + [\beta Y_0(\beta) - 2\alpha Y_1(\beta)]^2$; $J_0(\beta)$ is the Bessel function of the first kind of order zero; $J_1(\beta)$ is the Bessel function of the first kind of order one; $Y_0(\beta)$ is the Bessel function of the second kind of order zero; $Y_1(\beta)$ is the Bessel function of the second kind of order one; and α is S multiplied by the square of the ratio of the radius of the well within the water-yielding unit (r_w) to the radius of the casing within which water levels are measured. (In all tests analyzed for this study, α = S because the well radius in bedrock is the same as the casing radius.) Papadopulos and Cooper (1967) solved Equation 2 using numerical integration, and presented tabulated values of F(u, α) for selected values of u and α.

Picking (1994) defined Y_p as the drawdown in the pumped well at the instant pumping stopped, t_p as the duration of pumping, and Y_g as the residual drawdown at any time t_g measured since pumping stopped. Accordingly,

$$Y_p = \frac{Q}{4\pi T}W(p)$$

where

$$p = \frac{r_w^2 S}{4\,T\,t_p} \tag{3}$$

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and

$$Y_g = \frac{Q}{4\pi T}[W(z) - W(g)]$$

where $$g = \frac{r_w^2 S}{4\,T\,t_g}$$

$$z = \frac{r_w^2 S}{4\,T\,(t_p + t_g)} \qquad (4)$$

Dividing Equation 4 by Equation 3, Picking obtained the following dimensionless expression:

$$Y_g/Y_p = [W(z)–W(g)]/W(p) \qquad (5)$$

The terms W(p), W(g), and W(z) refer to particular values of F(u,α) for the particular times defined in Equations 3 and 4. Picking developed (but did not publish) a computer program that would plot Y_g/Y_p against t_g for specified values of α, measured values of t_p and r_w, estimated trial values of T, and values of F(u,α) interpolated from the tables of Papadopulos and Cooper (1967).

Klusman (1999) wrote a computer program in Fortran that replicates Picking's method, except that instead of interpolating within a table to obtain values of F(u,α) needed to solve Equation 5, it directly calculates values of F(u,α) for specified well, aquifer, and test properties. The adaptive trapezoidal method is used to integrate the function in Equation 2. The program, termed PICKINGmodel, is listed in Appendix A of Klusman (1999).

Application

The PICKINGmodel program is menu-driven. The user is prompted to enter the name chosen for an output file (prompt 1), the name of a preset input file containing trial values of α (prompt 3), the radius of the well (prompt 4), the duration of pumping (t_p) in seconds (prompt 5), maximum recovery time (t_g) for which computation is desired, which is ordinarily equal to the time span of the field data set, in seconds (prompt 6), and a trial value of transmissivity (prompt 7). A blank line must be included at the end of each alpha file used as input. The program will then define a type curve for each α value by calculating values of [W(z)–W(g)]/W(p) corresponding to many values of t_g, after which the user closes the output file (prompt 2) and ends the program (prompt 8). The resulting type curves can be plotted (manually or by a computer graphics package) on the same scale as the array of field data, in which Y_g/Y_p is plotted against t_g. The user then selects new trial values of T and α as appropriate to create a type curve that more closely matches the data array. A lower T moves all type curves to the right; a lower α yields a type curve that is steeper as well as moved to the right. The time required to calculate a batch of five type curves ranged from 5 min if all α values exceeded 1E-2 to 6 h if α values as small as 1E-7 were included. Two typical examples of type curves fitted to data curves are shown in Figure 1.

The relation of transmissivity as computed by PICKINGmodel to specific capacity as observed during the first 30 to 60 s of recovery (after ~1 min of pumping) is shown in Figure 2. Regression analysis of this relation yielded the following:

$$\log(\text{transmissivity, in m}^2/\text{d}) = -0.1835 + 1.118\log(\text{specific capacity, in L/min/m}) \quad R^2 = 0.87 \qquad (6)$$

Because specific capacity is easily calculated from test measurements, Equation 6 may prove useful in selecting an initial trial value of transmissivity to launch PICKINGmodel.

Accuracy and Limitations

PICKINGmodel generates mutually consistent type curves for values of α ranging from 1 to 1E-7. Larger values of α (which are physically implausible) result in type curves inconsistent with the normal pattern of curve displacement to the left as α increases. Smaller values of α (which are unlikely in aquifers) result in incomplete, unusable type curves.

Picking (1994) noted that as duration of pumping becomes very small, his computational procedure should generate type curves that approach those of a slug test. He plotted three type curves for very small values of dimensionless time (Tt_p/r^2 = 1E-3) on the same graph as points calculated by Cooper Jr. et al. (1967) for an instantaneous slug-test response. Picking observed that values of Tt_g/r^2 generated by his procedure were 20% smaller than those calculated by Cooper Jr. et al. (1967) and attributed the discrepancy to use of linear interpolation between the W(u) values in his lookup table. Figure 3 (which was prepared in exactly the same manner as Figure 3 of Picking [1994]) shows that PICKINGmodel duplicates the slug test values calculated by Cooper Jr. et al. (1967) for α = 1E-3 and 1E-5, and deviates only slightly from their values for α = 1E-1. This result indicates that, as suggested by Picking (1994), replacement of the interpolation procedure with calculated values of the exponential equation in PICKINGmodel has virtually eliminated this source of error at short pumping times.

The principal limitations of PICKINGmodel are those common to all brief single-well tests. (1) Results apply only to a small volume around each well. If median aquifer properties and pumping data from this study are inserted in the Theis solution (Freeze and Cherry 1979), a drawdown of 3 mm is predicted at a radial distance of only 4.25 m from the pumped well. (2) Type curves for successive α values are so similar that data sets commonly can be matched reasonably well to type curves that differ in α by one or two orders of magnitude. Therefore, storativity estimates should not be considered reliable.

In this study, PICKINGmodel was applied to 25 brief tests of domestic wells penetrating shale or fine sandstone bedrock. Depth to water was measured frequently during 0.5 to 2 min of pumping and several hours of recovery. All tests were analyzed not only by PICKINGmodel, but also by the widely used slug test procedure of Cooper Jr. et al. (1967), which can be applied to recovery after brief pumping by extrapolating the test data to estimate the time and magnitude of an equivalent slug; 13 tests were analyzed by

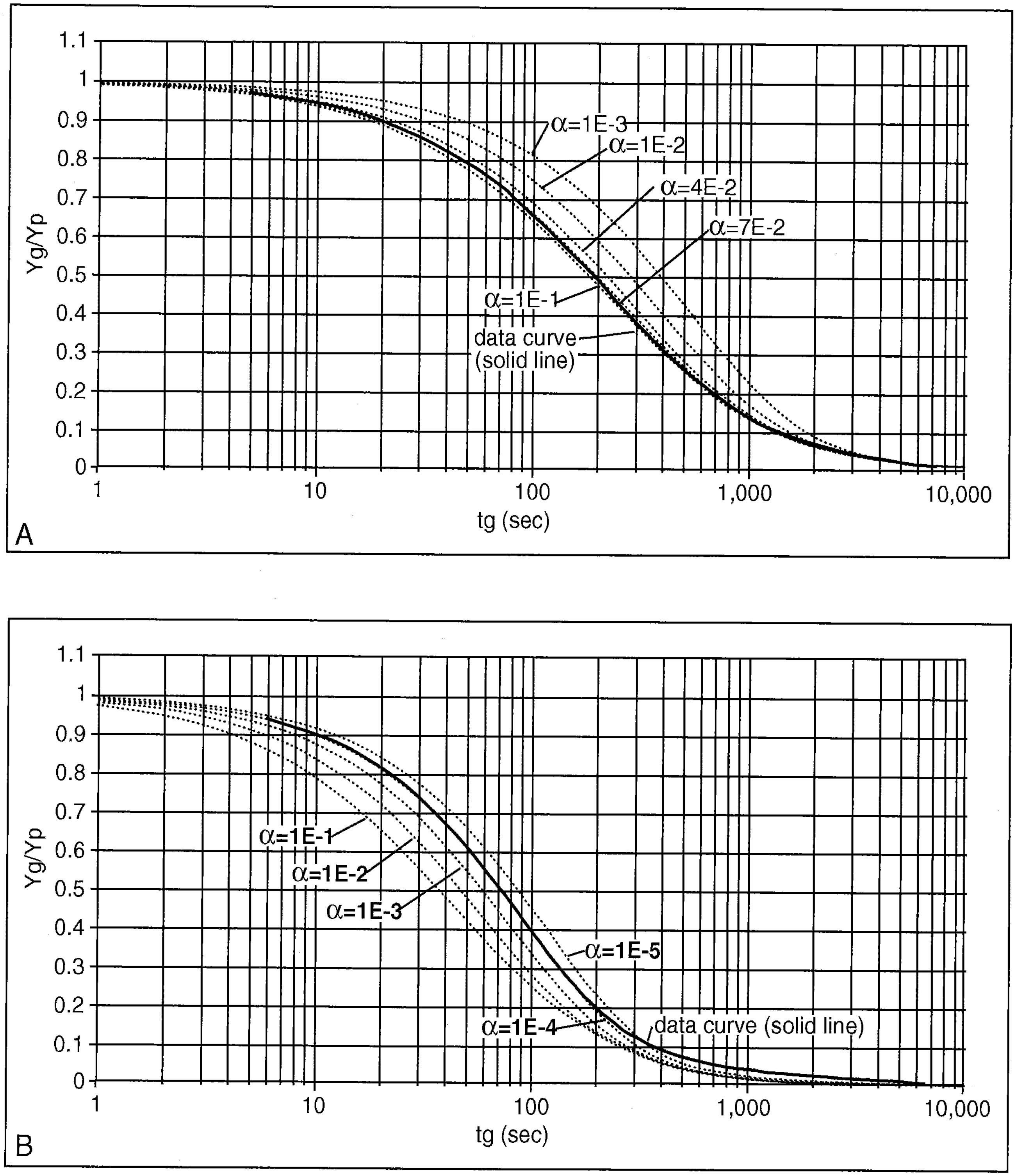

Figure 1. Data curves from two pumping tests fitted to sets of custom type curves generated by PICKINGmodel. (a) Test 115, transmissivity = 0.2 E-4 m^2/s (1.8 m^2/d). (b) Test 106, transmissivity = 0.15 E-3 m^2/s (14 m^2/d).

the method of Mishra and Chachadi (1985). All three methods yielded similar estimates of transmissivity; regression of log-transformed values from one method against another had coefficients of determination (R^2) of 0.94 or better.

Test data matched a PICKINGmodel type curve quite closely, as in Figure 1a, for at least several thousand seconds in 10 of the 25 wells analyzed by this method. More commonly, however, the data points systematically rose above or fell below the type curve after the first 20 to 400 s of recovery (Figure 1b). These departures may reflect what would be termed boundaries in classic image well analysis (Ferris et al. 1962). In other words, data points above the type curve imply anomalously slow recovery, which could result if one or more productive fractures intersected the well, but pinched out, narrowed, or blended into a network of lesser fractures some distance from the well, as documented by comprehensive tests at a site in Connecticut (Gernand and Heidtman 1997). Karasaki et al. (1988) illustrated several type curves that were custom designed to represent slug test responses to different idealized fracture

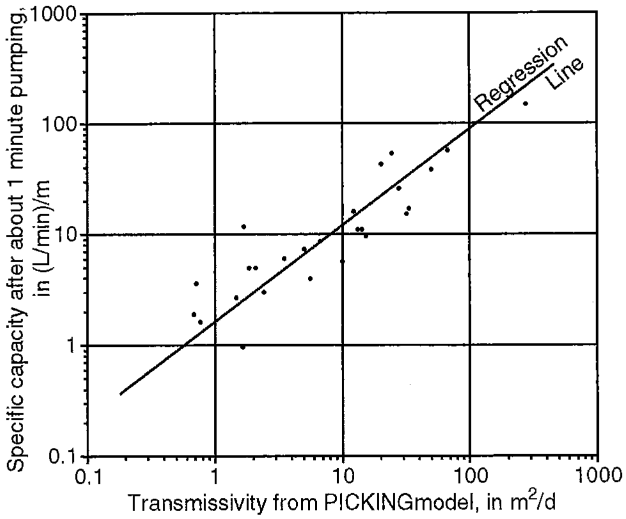

Figure 2. Relation of transmissivity from PICKINGmodel to specific capacity during the first 30 to 60 s of recovery, following ~1 min of pumping.

geometries, which suggests that it may be possible to design an iterative process that could modify type curves to match a variety of observed data distributions. Alternatively, if brief tests of multiple wells in some locality are to be used to characterize aquifer transmissivity, some adjustment of test results to allow for apparent boundary effects may be advisable.

Availability of Computer Program

The source code for PICKINGmodel, an executable version compiled on a PC using Windows NT4.0 operating system and that also runs under WINXP and WIN2K, sample output from one run, and an Excel spreadsheet folder containing all field data and model outputs for one test may be downloaded from the U.S. Geological Survey's New York District Web site at *http://ny.usgs.gov/projects/pickingmodel*. The Web site links to an open-file report by Randall and Klusman (2004) that includes a rationale for brief pumping tests of domestic wells, a description of field procedures for those tests, and transmissivity values calculated for individual wells by different methods.

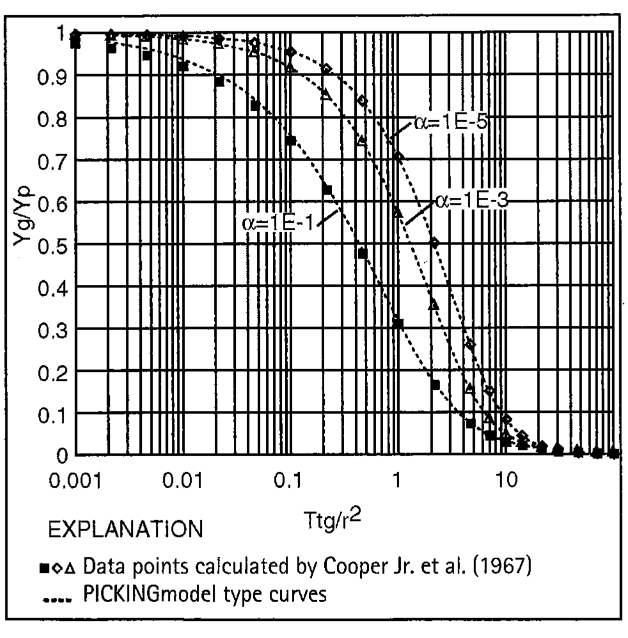

Figure 3. Comparison of type curves generated by PICKINGmodel for very short pumping time (Tt_p/r^2 = 1E-3) with slug test response as calculated by Cooper Jr. et al. (1967).

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